

Inflationary Tensor Perturbations After BICEP

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The measurement of B-mode polarization of the cosmic microwave background at large angular scales by the BICEP experiment suggests a stochastic gravitational wave background from early-universe inflation with a surprisingly large amplitude. The power spectrum of these tensor perturbations can be probed both with further measurements of the microwave background polarization at smaller scales, and also directly via interferometry in space. We show that sufficiently sensitive high-resolution B-mode measurements will ultimately have the ability to test the inflationary consistency relation between the amplitude and spectrum of the tensor perturbations, confirming their inflationary origin. Additionally, a precise B-mode measurement of the tensor spectrum will predict the tensor amplitude on solar system scales to 20% accuracy for an exact power law tensor spectrum, so a direct detection will then measure the running of the tensor spectral index to high precision.

The remarkable observations of the BICEP experiment [1], if correct, may reveal the existence of tensor perturbations in the universe with an unexpectedly large amplitude. The measured B-mode component of the polarization power spectrum [2, 3] is consistent with a scale-invariant gravitational wave background with a tensor-scalar ratio of $r = 0.2$. These tensor perturbations could arise from inflation in the early universe, but further characterization of the signal is needed to make this case compelling. The large amplitude of the signal creates a realistic possibility for two independent tests of an inflationary origin for the tensor perturbations: one, higher precision measurements of the B-mode polarization, and two, the direct detection of the gravitational wave background with space-based interferometry. Previous work has considered this pairing of experiments as an inflation probe [4–10], but their combination becomes far more informative if the amplitude of the tensor perturbations is as large as $r = 0.2$.

Inflation generally produces a power-law power spectrum for both scalar and tensor perturbations, $P_S(k) = A_S(k/k_0)^{n_s-1}$ and $P_T(k) = A_T(k/k_0)^{n_T}$ (see, e.g., the classic review [11]). The scalar perturbation amplitude A_S and spectral index n_s have been determined to high precision through measurements of the microwave background temperature anisotropies [12–14]. Thus the amplitude of tensor perturbations is generally characterized by the tensor-scalar ratio, $r \equiv P_T/P_S$, evaluated at the fiducial wavenumber $k_0 \equiv 0.0014 \text{ Mpc}^{-1}$.

The simplest models of inflation, which involve a single dynamical degree of freedom evolving slowly compared to the expansion rate of the universe (single field, slow-roll models) predict a relation between the tensor-to-scalar ratio and the tensor power law index known as the *consistency relation*, $r = -8n_T$ [15]. This connection arises because both the tensor and scalar power spectrum arise from the single degree of freedom. If tensor perturbations with $r = 0.2$ are generated by inflation, the naive expectation is $n_T = -0.025$.

This value for n_T will be observable with anticipated microwave background polarization experiments. Our ability to measure n_T is limited by cosmic variance, which provides a fundamental limit to how well the tensor power spectrum can be measured: we only have a single sky to measure. The cosmic variance of the B-mode polarization power spectrum multipole C_l^{BB} is approximately $\sigma_l = \sqrt{2/(2l+1)f_{\text{sky}}} C_l^{BB}$, where f_{sky} is the fraction of the full sky mapped by a given experiment. In addition to tensor perturbations, gravitational lensing of the larger E-mode polarization component will produce B-mode polarization contributing to this cosmic variance [16]. However, with sufficiently sensitive high-resolution polarization maps, the lensing signal can be measured directly using the characteristic non-gaussian distribution of polarization which lensing creates. Knox and Song [17] originally estimated how well the polarization field can be “delensed” using a quadratic maximum-likelihood estimator of Hu and Okamoto [18], finding cosmic variance due to the residual lensing signal of roughly 10% for a perfect sky map. Subsequent work by Hirata and Seljak [19] demonstrated that an iterative application of the quadratic estimator can push delensing significantly further, given maps of sufficient sensitivity and angular resolution. The ability of an experiment with very low noise to measure n_T from a B-mode polarization map will be determined by the cosmic variance from the sum of the primordial tensor signal plus the residual lensing signal after any delensing procedure.

Here we estimate the ability of several nominal future polarization experiments to constrain n_T and test the consistency relation (see also [20]). Sensitivity and angular resolutions of these experiments are given in Table 1; for all cases we assume a sky coverage of $f_{\text{sky}} = 0.5$, corresponding to a single ground-based experiment. Here we assume no foreground contamination or systematic errors; to make these assumptions more believable for a ground-based experiment, we only consider power spectrum measurements with $l > 50$, corresponding to angular scales smaller than 4 degrees. We also assume the theoretical power spectrum of gravitational lensing is known exactly, which will be a good assumption for upcoming experiments based on our knowledge of linear structure growth in the standard

cosmological model. Uncertainties in the lensing model (currently around 2% in the lensing amplitude, e.g. Fig. 12 of [1]) will only cause small changes to these results. We assume the residual lensing signal amplitude given by Seljak and Hirata [19] (listed in Table 1) for the given map sensitivities and angular resolution.

Experiment	$\sigma_{Q,U}$ ($\mu\text{K-arcmin}$)	beam (FWHM) (arcmin)	lensing residual [19]
Example A	1.41	4	10%
Example B	0.5	4	5%
Example C	0.25	4	1%

TABLE I: Parameters of model polarization experiments.

Figure 1 displays allowed values for each experiment in the r - n_T plane, for a fiducial model with $r = 0.2$ and $n_T = -0.025$ satisfying the inflationary consistency relation. These have been calculated using a simple quadratic likelihood evaluated on a grid of models in the parameter plane. We compute C_l^{BB} using the CAMB package [21] and use only multipoles $50 < l < 2000$ in computing the likelihoods, with a pivot scale $k_0 = 0.002 \text{ h Mpc}^{-1}$. Including higher multipoles has a negligible effect in the likelihoods; improving angular resolution to 2 arcminutes will decrease the lensing residual marginally, by roughly 10% (see Fig. 5 in Ref. [19]). Constraints on n_T improve dramatically as the map sensitivity increases from $1.4 \mu\text{K-arcmin}$ to $0.25 \mu\text{K-arcmin}$, due to better delensing. At the lower sensitivity value, given by Example C, n_T has a normal error of around 0.006, so is measured to be different from 0 at around 4σ . With perfect cleaning of lensing, the significance away from 0 for Example C would increase marginally from 4σ to 5σ , but improving on the residual lensing contribution in Example C requires a more sophisticated treatment of delensing [19]. The same sensitivity and resolution for a full-sky map, presumably from a satellite mission, would increase sky coverage by a factor of 2, decreasing the cosmic-variance limited errors by a factor of $\sqrt{2}$, and give a normal error on n_T of around 0.004, constraining n_T away from zero at a 6σ level.

If the actual amplitude of the tensor perturbations turns out to be $r = 0.1$ instead of $r = 0.2$ (consistent with an alternate foreground dust model in [1]), then the consistency-relation value of n_T decreases by a factor of 2, while the B-mode signal used to measure n_T also decreases in amplitude by a factor of 2. Then a full-sky map with the sensitivity of Example C provides a determination of n_T with error around 0.006, which is now different from 0 at a 2σ significance. The consistency relation still passes a strong test, but we are not able to distinguish between a consistency-relation inflation model and a naive scale-invariant tensor background of unspecified origin.

The example experiments in Table 1 represent a range encompassing possible sensitivities for a so-called “Stage 4” microwave background experiment [22, 23]. The ability to measure n_T depends strongly on the sensitivity in this range. Polarization maps with 4 arcminute resolution or better and map sensitivity of $0.25 \mu\text{K-arcmin}$ or better can decisively test the inflationary consistency relation between r and n_T for the BICEP value of $r = 0.2$. A measurement of n_T obeying the consistency relation and inconsistent with the generic scale-invariant spectrum $n_T = 0$ would provide highly non-trivial evidence in favor of the tensor perturbations arising from a simple single-field, slow-roll inflation epoch in the early universe. Note that a value of n_T different from the inflation consistency relation would not necessarily rule out inflation as the source of the tensor perturbations, but could alternately give valuable information that the inflation mechanism was more complicated than a simple slow-roll model, provided the tensor perturbations did arise from inflation.

The second test of inflation is a direct detection of the tensor perturbations using space-based interferometry. A stochastic background of gravitational waves with a scale-invariant power-law spectrum and an amplitude of $r = 0.2$ results in a tensor energy density of $\Omega_{\text{GW}} h^2 \simeq 10^{-15}$. Fortunately, this tensor amplitude is about the value which maximizes the direct detection amplitude at terrestrial scales [4, 24]. NASA’s Big Bang Observer (BBO) concept study [25], an extension of the Laser Interferometer Space Antenna (LISA) proposal to higher sensitivities and shorter satellite separations, would detect this signal at a frequency of 0.1 Hz with a significance of 100σ with one year of observing [26, 27], in the absence of confusion noise from white-dwarf binaries at cosmological distances [27]. Such a binary background will be isotropic with a steeply falling power spectrum; a nonisotropic binary contribution from the Milky Way would also need to be accounted for. A second more speculative stochastic gravitational wave contribution could arise from a possible early-universe phase transition at the electroweak scale [28, 29]. For high-precision characterization of the primordial signal, the competing backgrounds would either need to be modeled with comparable precision, or interferometric measurements would need to have a low-frequency cutoff, reducing the detection significance for BBO substantially [26]. Experiments with greater sensitivity and higher frequency ranges than BBO have been contemplated: Kudoh et al. [26] have calculated that with an 0.2 Hz lower frequency cutoff, their “Fabry-Perot DECIGO” [30, 31] would detect the $r = 0.2$ gravitational wave background at 10σ , while their “Ultimate DECIGO” [32] would detect it at $5 \times 10^4 \sigma$.

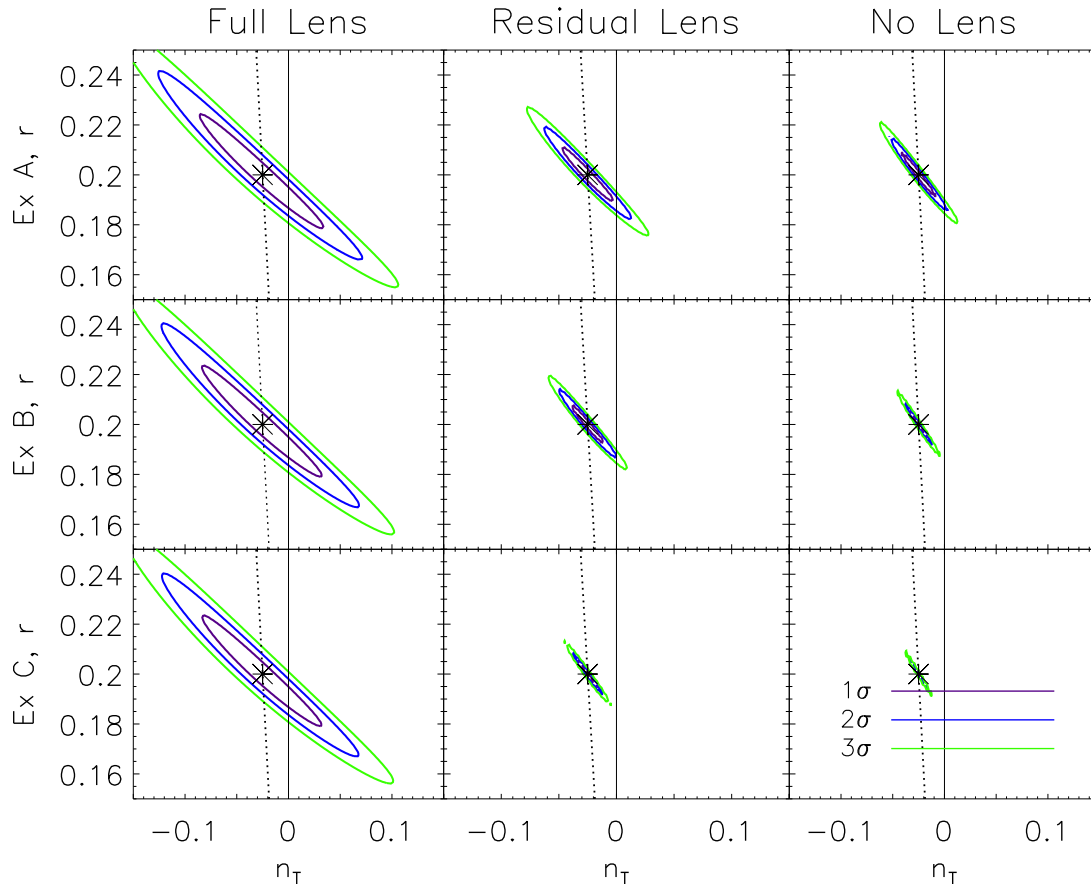


FIG. 1: Likelihood countours in the r - n_T plane for the model polarization experiments in Table 1. The fiducial model is $r = 0.2$ and $n_T = -0.025$, indicated by *. The dotted line indicates the inflation consistency relation. The vertical line indicates $n_T = 0$ for reference. (Left) Full lensing contribution to the cosmic variance error. (Center) Residual lensing contribution after delensing (see Table 1 for residual lensing noise levels), and (Right) No lensing contribution to the cosmic variance, for comparison.

The B-mode polarization of the microwave background arises from tensor modes with a characteristic wavelength of $k_0^{-1} \simeq 500$ Mpc, while direct detection experiments probe characteristic wavelengths of $c/\nu = 2 \times 10^{-2}$ A.U., a range covering a factor of 10^{16} in wavelength. A determination of r and n_T in the B-mode power spectrum means that the tensor spectrum can be extrapolated to smaller wavelengths, assuming a perfect power law spectrum. The amplitude at a smaller scale will have an uncertainty governed by the uncertainties in r and n_T at the larger scale, like those displayed in Fig. 1. If $r = 0.2$, a full-sky B-polarization map with sensitivity below $1 \mu\text{K arcmin}$ will determine $n_T = -0.025$ with an error of around 0.004. Then extrapolating to a scale 10^{16} times smaller in wavelength using two different values of n_T differing by 0.004 gives an amplitude difference of 20%. This is much larger than the difference due to uncertainty in r which we can ignore.

A direct detection experiment which could measure the tensor amplitude to significantly better than 20% could thus detect a difference from the predicted amplitude with an uncertainty of around 20% of the measured amplitude. Such a difference would arise if the spectrum is not a perfect power law, but rather has some variation in its power law with scale. In analogy with the running of the density perturbation spectrum [33], define the running of the tensor spectral index $\alpha_T \equiv dn_T/d\ln k$. A value of $\alpha_T = 2 \times 10^{-4}$ will result in a difference in amplitude of 20% when extrapolated over a factor of 10^{16} in wavelength; so by comparing with the values of r and n_T measured from B-mode polarization, a direct measurement of the tensor amplitude with interferometry can measure α_T with an error of around 2×10^{-4} . (If $r = 0.1$, then the error on running becomes weaker by about factor of 2.) Discussions of tensor perturbations until now have commonly claimed that a significant measurement of α_T is hopeless – but we see that it is likely possible to

measure the tensor running *better* than scalar running, *if* precise measurements of both r and n_T are obtained from B-mode polarization.

Direct detection of the gravitational wave background with a very high significance, by some future experiment like Ultimate DECIGO, would allow a second measurement of n_T , at a wavelength of around 0.02 A.U. Then this value could be compared with the spectrum extrapolated from the B-mode n_T plus the running required to give the measured amplitude at A.U. scales. Consistency would demonstrate that a constant-running approximation to the tensor power spectrum is valid, and further verify the inflationary origin of this background; in this case, we can hope to have six measured quantities characterizing the physics of inflation (the amplitude, power law index, and running for both scalar and tensor perturbations). If the two values do not match, it would show that further parameters aside from a spectral index and a constant running are required to adequately describe the tensor power spectrum.

Inflation was invented to solve a well-known litany of cosmological conundrums: the observed flatness and isotropy of the universe and the absence of magnetic monopoles. Inflation also provides a mechanism for generating a primordial gaussian random distribution of nearly scale-invariant and phase-coherent density perturbations, which gives an excellent match to observed microwave background temperature anisotropies and the large-scale distribution of galaxies. Inflation additionally makes a completely generic prediction of a nearly scale-invariant background of tensor perturbations, which once generated will propagate unimpeded until the present day, and unchanged save for their dilution and stretching due to the expansion of the universe. But the amplitude of this stochastic background depends on the unknown energy scale of inflation. Prior to St. Patrick's Day 2014, we had only suggestive and somewhat controversial theoretical reasons that the tensor amplitude was large enough ever to detect. If the B-mode polarization measured by BICEP is due to inflationary tensor perturbations, their amplitude is a factor of 10 to 100 larger than cautious optimists had hoped. As a result, we suddenly and unexpectedly have within our reach the chance to probe the unknown physics governing the universe at an age of 10^{-36} seconds and an energy scale of 10^{16} GeV, with two completely different experimental approaches. We are surely obliged to try.

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